But for low pressure field was assumed: $U \cong E_r/B_z$; so Eq. (10) becomes:

$$U/2 + \frac{j_{er}E_r}{m_iU(\delta\dot{n}/\delta z)} = \epsilon_{d_1i}/(m_iU)$$
$$U^2 = 2\left[\epsilon_{d_1i}/m_i - \frac{j_{er}E_r}{m_i(\delta\dot{n}/\delta z)}\right] \quad (11)$$

The expression (11) for ion velocity U differs from the corresponding expression obtained by Patrick and Schneiderman[1] for the term $-j_{er}E_r/[m_i(\delta\dot{n}/\delta z)]$. The radial component of the current density j_{er} due to electron motion [Eq. (9)] is strongly affected by the orthogonal magnetic field and its contribution to the total current j_r can be less than the ion radial current j_{ir}. From the data reported by Fahleson[3] on the existence of a critical velocity v_c of ions, and following the calculations performed by Alfvén,[2] a difference of some electron volts is found between the measured and the calculated ion energy. This difference shows that the measured value of v_c is less than the calculated value, taking into account the total current density in both the momentum and the energy equations. So could be justified the existence of the term $-j_{er}E_r/[m_i(\delta\dot{n}/\delta z)]$ in Eq. (11) obtained with ion current in the momentum equation (4).

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Reply by Authors to R. Giovanelli

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The argument presented by the authors was meant only to illustrate the kinds of arguments that are used in discussing the critical velocity, and was intended only to make plausible the appearance of a critical velocity and indicate the relevance of the original work on this subject by Alfvén and others. The subject of the critical veloicity is by no means closed, and a large literature[1–22] has evolved, based upon either the macroscopic balances of the kind used by Petschek,[1] or more microscopic effects involving the electron energy balance as used by Drobyshevsky.[2] The approach presented by the authors has been further amplified by Lin.[3] However, the basic assumption of this approach is that the coupling to the neutrals is small and neutral effects do not play an important role in the ionized components, energy, and momentum balance. This assumption is not agreed to by others, including Lehnert[4] and Block.[5] Lin shows that acceptance of this assumption implies that the ions, because of their larger mass, must take up the momentum caused by the $j \times B$ forces, although the electrons can absorb some of the energy. This leads to the use of the total current in the ion momentum equation but with an electron energy term included in the ion energy equation.

It is not clear how this work relates to the effect pointed out in the aforementioned technical comment. However, this approach in general is valid only for weak coupling between the ions and the neutrals $\omega_i\tau_{in} \gg 1$, in which case the electron current is negligible [see previously mentioned Eq.(9)], and the change in the critical velocity pointed out in the comment goes to zero.

Because the basic assumptions on which the whole approach is based are subject to question and because granted that the assumptions are correct, the change in critical velocity vanishes, the authors believe that the work in this note is not significant to the problem of the critical velocity.

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